Physics 846: Problem Set 1

Dr. Stroud

Due Thursday, January 14 at 11:59 P. M.

Note: Each problem is worth 10 points unless otherwise specified

1. The energy of a particular system, of one mole, is given by

\[ U = AP^2V \]  

where A is a positive constant of dimensions \([P]^{-1}\). Find the equation of the adiabats in the P-V plane.

2. (30 pts.) The following ten equations are purported to be fundamental equations of various thermodynamic systems. However, five are inconsistent with one or more of postulates II, III, or IV (see below) and are consequently not physically acceptable. In each case, sketch the fundamental relation between S and U (with N and V constant). Find the five equations that are not physically permissible, and indicate the postulates violated by each.

The quantities \(v_0\), \(\theta\), and \(R\) are positive constants, and in all cases in which fractional exponents appear, only the real positive root is to be taken.

(a). \( S = \left( \frac{R^2}{v_0^2} \right)^{1/3} \left( NVU \right)^{1/3} \)

(b). \( S = \left( \frac{R}{\theta^2} \right)^{1/3} \left( \frac{NU}{V} \right)^{2/3} \)

(c). \( S = \left( \frac{R}{\theta} \right)^{1/2} \left( NU + \frac{R\theta v_0^2}{v_0^2} \right)^{1/2} \)

(d). \( S = \left( \frac{R^2\theta}{v_0^2} \right) \frac{V^3}{(NU)} \)

(e). \( S = \left( \frac{R^2}{v_0^2} \right)^{1/5} \left[ N^2VU \right]^{1/5} \)

(f). \( S = NR \ln[UV/(N^2R\theta v_0)] \)
(g). \[ S = \left( \frac{R}{\theta} \right)^{1/2} (NU)^{1/2} \exp\left[-V^2/(2N^2v_0^2)\right] \]

(h). \[ S = \left( \frac{R}{\theta} \right)^{1/2} (NU)^{1/2} \exp\left(-\frac{UV}{N\theta v_0}\right) \]

(i). \[ U = \left( \frac{m\theta}{R} \right) \frac{S^2}{V} \exp[S/(NR)] \]

(j). \[ U = \left( \frac{R\theta}{v_0} \right) NV \left(1 + \frac{S}{NR}\right) \exp[-S/(NR)] \]

For reference, the four postulates as given by Callen are:

**Postulate I.** There exist particular states (called equilibrium states) of simple systems that, macroscopically, are characterized completely by the internal energy \( U \), the volume \( V \), and the mole numbers \( N_1 \), \( N_2 \),...\( N_r \) of the chemical components.

**Postulate II.** There exists a function (called the entropy \( S \)) of the extensive parameters of any composite system, defined for all equilibrium states and having the following property: The values assumed by the extensive parameters in the absence of an internal constraint are those that maximize the entropy over the manifold of constrained equilibrium states.

**Postulate III.** The entropy of a composite system is additive over the constituent subsystems. The entropy is continuous and differentiable, and is a monotonically increasing function of the energy.

**Postulate IV.** The entropy of any system vanishes in the state for which
\[
\left( \frac{\partial U}{\partial S} \right)_{V,N_1,N_2,\ldots,N_r} = 0
\]

(that is, at the zero of temperature).

3. (15 pts.) Find the three equations of state for a system with the fundamental relation
\[ \frac{S}{R} = \frac{UV}{N} - \frac{N^3}{UV}. \]

(a). Show that the equations of state in entropy representation are homogeneous zero-order functions

(b). Show that the temperature is intrinsically positive.

(c). Find the “mechanical equation of state” \( P(T,v) \).
(d). Find the form of the adiabats in the $P - v$ plane. (An “adiabat” is a locus of constant entropy, also called an “isentrope”.)