1) (5pts) Three charges are at the corners of a square (with side = 1 cm), as shown in the figure. What is the potential at the triangle corner W? Assume that $V(\infty)=0$.

$$V = \frac{4 \times 10^{-6} C}{4\pi\varepsilon_0(0.01m)} + \frac{4 \times 10^{-6} C}{4\pi\varepsilon_0(0.01m)} - \frac{4 \times 10^{-6} C}{4\pi\varepsilon_0(0.014m)} = 4.6 \times 10^6 V$$

2) (10 pts) What is the magnitude and direction of the electric field at W?

$$E_x = \frac{-4 \times 10^{-6} C}{4\pi\varepsilon_0(0.01m)^2} + \frac{4 \times 10^{-6} C(\cos 45)}{4\pi\varepsilon_0(0.014m)^2} = -2.3 \times 10^8 N/C$$

$$E_y = \frac{-4 \times 10^{-6} C}{4\pi\varepsilon_0(0.01m)^2} + \frac{4 \times 10^{-6} C(\sin 45)}{4\pi\varepsilon_0(0.014m)^2} = -2.3 \times 10^8 N/C$$

$$E = \sqrt{E_x^2 + E_y^2} = 3.3 \times 10^8 N/C, \quad \text{direction} = 225^0$$

Note: you didn’t have to calculate $E_x$, $E_y$ to figure out the direction of E. Its direction can be found from symmetry.

3) (10 pts) A point charge of $+5 \times 10^{-6}$ C is put at W. What is the magnitude and direction of the force on this charge?

$$\overline{F} = q \overline{E} \quad \text{Force is in same direction as E.}$$

$$|\overline{F}| = q |\overline{E}| = (5 \times 10^{-6} C)(3.3 \times 10^8 N/C) = 1.7 \times 10^3 N$$
4) A non-conducting sphere (with radius \( B \)) is inside two hollow conducting spheres as shown in the figure. All of the spheres are concentric. A total charge of +Q is uniformly distributed through the volume of the non-conducting sphere.

The inner conducting sphere (\( C < r < D \)) has excess charge \(-2Q\), the outer conducting sphere (\( E < r < F \)) excess charge \(3Q\) has on it.

Use GAUSS' LAW to find the following:

i) (5 pts) The charges on the inner and outer surfaces of the conducting spheres.

\[ \begin{align*}
\text{i) } & \text{CD inner: -Q, CD outer=-Q} \\
& \text{EF inner=+Q, EF Outer=+2Q}
\end{align*} \]

\( i\)  \( E = \frac{Qr}{4\pi \varepsilon_0 B^3} \)

\( iii\)  \( E = \frac{-Q}{4\pi \varepsilon_0 r^2} \)

\( iv\)  \( E = \frac{2Q}{4\pi \varepsilon_0 r^2} \)

5) (5 pts) A cylinder (of length 10 cm, radius 2 cm) contains 5 electrons inside of it. What is the net electric flux through the cylinder?

\[ \Phi = \frac{q_{\text{enc}}}{\varepsilon_0} = \frac{(5)(-1.6 \times 10^{-19} \text{C})}{8.85 \times 10^{-12} \text{ C}^2 / \text{Nm}^2} = -9.0 \times 10^{-8} \text{Nm}^2 / \text{C}^2 \]
6) In the following problem the equipotentials are concentric circles spaced by 10 cm. The innermost equipotential is at -20V, the outermost -140V.

a) (10 pts) What is the magnitude and direction of the electric field at point a?
b) (5 pts) Charge is moved from a to b and then from b to c and then from c to a. What is the total work done on this trip?

\[ |E| = \frac{\Delta V}{\Delta x} = \frac{40V}{0.1m} = 400V/m \]

direction of E

b) No work is done along a closed path

7) (5 pts) Which of the following could be the electrical potential (V) inside a solid conducting cylinder of length L with excess charge q a distance r away from its center?

\[ a) \ V = \text{constant} \quad b) \ V = \frac{q}{4\pi \varepsilon_0 r} \quad c) \ V = \frac{q \ln(r)}{4\pi \varepsilon_0 L} \quad d) \ V = \frac{\rho r^3}{4\pi \varepsilon_0 L} \]

8) (5 pts) Pick the correct statement:

a) The potential difference between points A and B in space is independent of the path taken to get from A to B.
b) If the electric field is zero at a location in space the potential must also be zero there.
c) If the potential is zero at a location in space, the electric field must also be zero there.
d) The potential is minus the derivative of the electric field.
e) All of the above are correct.
9) A non-conducting rod of length \( L = 0.4 \) cm has linear charge density \( +\lambda \). The total charge on the rod is \( Q = 3 \times 10^{-6} \) C.

Start with:

\[
E = \frac{1}{4\pi\varepsilon_0} \int dq \frac{1}{r^2}
\]

a) (5 pts) What is the direction (draw a line with an arrow) of the electric field at \( P \)?

\[
E
\]

b) (10 pts) Show that the magnitude of the electric field at \( P \) is given by:

\[
E = \frac{Q}{4\pi\varepsilon_0} \left( \frac{1}{a(L + a)} \right)
\]

This is very similar to HW problem 23-23 and the example done in lecture.

\[
E_x = \frac{1}{4\pi\varepsilon_0} \int_0^L \frac{\lambda dx}{(a + L - x)^2} = \frac{1}{4\pi\varepsilon_0} \left[ \frac{\lambda}{a + L - x} \right]_0^L = \frac{\lambda}{4\pi\varepsilon_0} \left( \frac{1}{a} - \frac{1}{L + a} \right) = \frac{Q}{4\pi\varepsilon_0} \frac{L}{(L + a) a} = \frac{Q}{4\pi\varepsilon_0} \frac{L}{a(L + a)}
\]

c) (5 pts) What is the value of the electric field when \( a = 2 \) m?

\[
E = 6.7 \times 10^3 \text{ N/C}
\]

d) (5 pts) Show the electric field of this line charge reduces to a point charge when \( a \gg L \).

If \( a \gg L \) then

\[
\frac{1}{L + a} = \frac{1}{a} \quad \text{then} \quad \frac{Q}{4\pi\varepsilon_0} \frac{1}{a(L + a)} = \frac{Q}{4\pi\varepsilon_0} \frac{1}{a^2}
\]