HONORS PHYSICS 5500  AU12  Exam 4 sample problems

1) Hydrogen Atom. \( H = \frac{p^2}{2m} - \frac{Ze}{r} \). The initial state is:
\[ |\psi(0)\rangle = \frac{1}{\sqrt{3}} |100\rangle + \frac{\exp(\sqrt{6})}{\sqrt{6}} |200\rangle + \frac{\sqrt{2}}{\sqrt{6}} |210\rangle. \]
At time \( t \) measurements are made. (a) What are the possible results and their probabilities for measurements of energy and angular momentum? (b) What is the probability that the electron's relative position will be found with \( x > 0 \)? (c) Same as (b) for \( z > 0 \). (d) What is the probability that \( r < a_0 \)? \( r < 2a_0 \)? (e) If at time \( t_1 \) we find \( r < a_0 \), collapsing the wave function, what is the probability that we will find \( E_1 \) when we measure energy? What about \( E_2 \)?

2) Hydrogenic Atom. \( H = \frac{p^2}{2m} - \frac{Ze}{r} \). A hydrogen atom \( (Z = 1) \) is in its ground state when a nuclear reaction at \( t = 0 \) causes the charge of the nucleus to change to \( Z = 2 \). At a later time \( t \) measurements are made. (a) What are the probabilities for finding the pair of values \( l \) and \( m \) if \( L^2 \) and \( L_\zeta \) are measured? (b) If energy is measure, what are \( P(E_1) \), \( P(E_2) \) and \( P(E_3) \)? In addition to an analytic expression, give a numerical approximation.

3) Harmonic Oscillator. \( H = \frac{p^2}{2m} + \frac{1}{2} m\omega^2 X^2 \). The initial state is:
\[ |\psi(0)\rangle = \frac{1}{\sqrt{3}} |0\rangle + \frac{1}{\sqrt{3}} |1\rangle - \frac{1}{\sqrt{3}} |2\rangle. \]
Measurements are made at time \( t \). (a) What are \( \langle H \rangle \), \( \langle X \rangle \) and \( \langle P \rangle \)? (b) What is \( \langle X^2 \rangle \)? Using \( \langle H \rangle \) and \( \langle X^2 \rangle \), compute \( \langle P^2 \rangle \). What is \( \Delta X \Delta P \)?

4) Harmonic Oscillator. \( H = \frac{p^2}{2m} + \frac{1}{2} m\omega^2 X^2 \). A system in the ground state is measured at time \( t = 0 \) and it is found that \( x > 0 \), collapsing the wave function. (a) What are \( P(n = 0) \), \( P(n = 1) \) and \( P(n = 2) \)? (b) What are \( \langle H \rangle \), \( \langle X \rangle \) and \( \langle P \rangle \)?