Write your name on the test booklet. Do NOT simply write an answer. Give a calculation and/or reasoning that supports your answer. Do all work and write all answers in the test booklet. Circle or clearly delineate all relevant work so that I do not take points off for errors in your scratch work.

1) Basic Math. Simplify numbers (e.g., write absolute values in terms of real numbers only, reduce answers with phases to expressions using sines and cosines) and normalize vectors where necessary.

(a) In a three-dimensional space with basis states, |1>, |2> and |3>, |φ> = i/3 |1> - (1 + i)/√2 |2> - e^{iπ/3}/2 |3>. What is <φ|φ>.

(b) $M = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$. Find the eigenvalues and eigenstates of $M$. Are the eigenstates complete?

(c) |ψ> = 1/√2 |+> - i e^{iφ}/√2 |->. Compute <ψ|Sz + Sy|ψ>.

2) A spin-1/2 particle is in the state, |ψ> = |+>_A. (a) If $S_y$ is measured, what are the possible results and their corresponding probabilities? (b) What is <S>, where S is the spin vector? (c) We first measure $S_x$ and then measure $S_y$. This gives four possible sequences of results, ++, +−, −+, −−. Compute the overall probability for each sequence.

3) The Hamiltonian for a spin-1/2 particle in a magnetic field that points in the x-direction can be written as $\hat{H} = \hbar \omega \sigma_x$ ($\sigma_z$ is given in notes).

(a) At $t = 0$, the z-component of spin is measured and yields $-\hbar/2$. What is |ψ(t)>?

(b) At $t = t_1$, the y-component of spin is measured. What are the possible results and the probabilities for each possible result?

(c) What is <S>?
\[ \hat{A} |a_n\rangle = a_n |a_n\rangle, \quad \langle a_m | a_n \rangle = \delta_{mn} \]
\[ 1 = \sum_n |a_n\rangle \langle a_n| \]

i) \( |\psi(+)\rangle \) - physical states represented by vectors

ii) \( \hat{A} \) - observation associated with Hermitian op.

iii) \( a_n \) - possible result are eigenvalues

iv) \( P(a_n) = |\langle a_n | \psi(+) \rangle|^2 \) - probability of \( a_n \)

v) \( i\hbar \frac{d}{dt} |\psi(+)\rangle = \hat{H} |\psi(+)\rangle \quad \text{or} \quad |\psi(t)\rangle = e^{-i\hat{H}t/\hbar} |\psi(0)\rangle 

\[ \hat{H} \neq \hat{A}(+) \]

\[ S_i = \frac{\hbar}{2} \sigma_i, \quad \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \]

\[ [S_x, S_y] = i\hbar S_z + \text{cyclic permutations} \]

\[ \hat{n} \cdot S |\psi(+)\rangle = \frac{\hbar}{2} |\psi(+)\rangle \]

\[ |\psi(+)\rangle = \cos \frac{\theta}{2} |+\rangle + \sin \frac{\theta}{2} e^{i\phi} |-\rangle \]

\[ \cos \frac{\theta}{2} |+\rangle - \cos \frac{\theta}{2} e^{i\phi} |-\rangle \]