Write your name on the test booklet. Do NOT simply write an answer. Give a calculation and/or reasoning that supports your answer. Do all work and write all answers in the test booklet. Circle or clearly delineate all relevant work so that I do not take points off for errors in your scratch work.

1) Infinite Square Well. \( V(x) = 0 \) when \(|x| < L/2\) and \( V(x) = \infty \) when \( x > L/2 \). \( H|n >= E_n|n > \) for \( n = 1, 2, 3, ... \) At \( t = 0 \):

\[
|\psi(0) >= \frac{i}{\sqrt{5}} |2 > -\sqrt{\frac{2}{5}} |3 > .
\]

(a) What is the probability of finding the particle in the right side of the box \((x > 0)\) at \( t = 0 \)?

(b) What is the average energy, \( < H > \), as a function of time?

(c) Compute the average position, \( < X > \), as a function of time.

2) Scattering. A particle with energy \( E = \frac{k^2}{2m} \) approaches a delta-function potential, \( V(x) = g\delta(x) \), from the right, from positive infinity. Compute \( R \) and \( T \), the probability that it will reflect and the probability that it will pass through the delta-function and be transmitted to \( x \to -\infty \).

3) The potential is infinite for \( x > 0 \), with a finite well next to this wall, \( V(x) = -V_0 \) for \(-L < x < 0 \). \( V(x) = 0 \) for \( x < -L \). (a) Under what conditions will a particle of mass \( m \) have two bound states? (b) What is the 2nd bound state? You do not need to determine the normalization constant, but the explicit form of the solution in each region must be given along with a graphical solution of the eigenvalue condition.

EXTRA CREDIT: \( V(x) = -g\delta(x) + g\delta(x - L) \). There is an attractive delta-function at \( x = 0 \) and a repulsive delta-function at \( x = L \). Particles with energy \( E = \frac{k^2}{2m} \) approach the potentials from the left. (a) What are the transmission and reflection probabilities, as functions of \( k \)? (b) Discuss the \( k \to 0 \) and \( k \to \infty \) limits. (c) Is it possible to adjust the strength, \( g \), and or separation between the potentials, \( L \), so that everything is reflected?
\[ \psi(t) = \exp[-iH(t-t_0)] \left| \psi(t_0) \right\rangle \]

\[ \hat{x} |x\rangle = x |x\rangle, \quad \langle x' | x \rangle = \delta(x-x') \]

\[ \hat{p} |k\rangle = k |k\rangle, \quad \langle k | k' \rangle = \delta(k-k') \]

\[ I = \int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} x' x dx' = \int_{-\infty}^{\infty} dk |k\rangle \langle k| \]

\[ \langle x | k \rangle = \frac{e^{ikx}}{\sqrt{2\pi}} \]

\[ \langle x | \psi(t) \rangle = \psi(x,t) \quad \langle k | \psi(t) \rangle = \hat{\psi}(k,t) \]

\[ \hat{H} = \frac{\hat{p}^2}{2m} + V(\hat{x}) \quad \hat{H} |\phi_n\rangle = E_n |\phi_n\rangle \]

For \( V(x) = V - \text{constant} \):

\[ \frac{d^2}{dx^2} \phi_n(x) = 2m (V - E_n) \phi_n \]

- If \( V - E_n < 0 \), let \( 2m (V - E_n) = -k_n^2 \)

\[ \Rightarrow \frac{d^2}{dx^2} \phi_n(x) = -k_n^2 \phi_n(x) \]

- If \( V - E_n > 0 \), let \( 2m (V - E_n) = -g_n^2 \)

\[ \frac{d^2}{dx^2} \phi_n(x) = g_n^2 \phi_n(x) \]
Infinite Sq. Well

\[ V(x) = \begin{cases} 0, & |x| \leq \frac{L}{2} \\ \infty, & |x| > \frac{L}{2} \end{cases} \]

Eigenstates:

\[ \phi_n(x) = \sqrt{\frac{2}{L}} \cos \left( \frac{n \pi x}{L} \right) ; n = 1, 3, \ldots \]

\[ \phi_n(x) = \sqrt{\frac{2}{L}} \sin \left( \frac{n \pi x}{L} \right) ; n = 2, 4, \ldots \]

\[ E_n = n^2 E_1 , \quad E_1 = \frac{1}{2m} \left( \frac{\pi}{L} \right)^2 \]

\[ \mathbf{j} \mathbf{in} \quad \text{for} \quad A e^{i k x} , \quad j = |A|^2 \frac{k}{m} \]

\[ \mathbf{j}_{\text{in}} \quad \text{incoming current} \]

\[ \mathbf{j}_{\text{R}} \quad \text{reflected current} \]

\[ \mathbf{j}_{\text{T}} \quad \text{transmitted current} \]

\[ R = \frac{\mathbf{j}_{\text{R}}}{\mathbf{j}_{\text{in}}} \quad T = \frac{\mathbf{j}_{\text{T}}}{\mathbf{j}_{\text{in}}} \]
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\[ \int \cos(p \cdot x) \sin(q \cdot x) \, dx = \frac{x \cos[(p-q) x]}{2(p-q)} - \frac{\sin[(p-q) x]}{2(q-p)^2} \]

\[ - \frac{x \cos[(p+q) x]}{2(p+q)} + \frac{\sin[(p+q) x]}{2(q+p)^2} \]