Write your name on the test booklet. Do NOT simply write an answer. Give a calculation and/or reasoning that supports your answer. Do all work and write all answers in the test booklet. Circle or clearly delineate all relevant work so that I do not take points off for errors in your scratch work. Simplify numbers (e.g., write absolute values in terms of real numbers only, reduce answers with phases to expressions using sines and cosines) and normalize vectors where necessary.

1) **Infinite Square Well.** \( V(x) = 0 \) when \( |x| < L/2 \) and \( V(x) = \infty \) when \( |x| > L/2 \). \( H|n> = E_n|n> \) for \( n = 1, 2, 3, \ldots \) At \( t = 0 \):

\[ \psi(x, t = 0) = N(x + L/2)(x - L/2), \]
for \( |x| < L/2 \). At time \( t \) later, you make measurements. (a) What are the probabilities of finding: (i) \( E_1 \), (ii) \( E_2 \) and (iii) \( E_3 \) if you measure energy? (b) What is the average energy, \( <H> \), as a function of time? (c) Compute the average position, \( <X> \), as a function of time?

2) **Scattering.** \( V(x) = V_0 > 0 \) if \( x < 0 \) and \( V(x) = 0 \) if \( x > 0 \). A beam is shot in from the right (coming from \( +\infty \)) with \( E > V_0 \). Compute the transmission and reflection probabilities.

3) **Harmonic Oscillator.** \( H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2X^2 \). At \( t = 0 \) the state is:

\[ |\psi(0)\rangle = \frac{i}{\sqrt{2}} |0\rangle + \frac{1}{\sqrt{2}} |1\rangle - \frac{1}{\sqrt{2}} |2\rangle. \]

At time \( t \), measurements are made. (a) What is the average energy, \( <H> \)? (b) What are the average position, \( <X> \), and momentum, \( <P> \). Compute these and verify that they satisfy their classical relationship. (c) What are \( <X^2> \) and \( <P^2> \)? Verify \( \Delta x \Delta p \geq \frac{\hbar}{2} \).

**EXTRA CREDIT** (Warning: do not try until all else is complete.) \( V(x) = -V_0\theta(x + a/2)\theta(x - a/2) - \beta\delta(x + a/2) - \beta\delta(x - a/2) \). Assume everything is fixed except \( V_0 \). How large must \( V_0 \) be so that there are two bound states?