Write your name on the test booklet. Do NOT simply write an answer. Give a calculation and/or reasoning that supports your answer. Do all work and write all answers in the test booklet. Circle or clearly delineate all relevant work so that I do not take points off for errors in your scratch work.

1) Scattering. A particle with energy \( E = \frac{k^2}{2m} \) approaches the step potential drawn below from the right, from positive infinity. The potential \( V(x) = 0 \) for \( x > 0 \) and \( V(x) = -V_0 \) for \( x < 0 \), so the particle "sees" the potential step down. Compute \( R \) and \( T \), the probability that it will reflect and the probability that it will pass over the step and be transmitted to \( x \to -\infty \).

2) Harmonic Oscillator. (a) Compute \( \langle \hat{X} \hat{P} \rangle \) and \( \langle \hat{P} \hat{X} \rangle \) for a particle in harmonic oscillator state \( |n \rangle \). (b) Compute \( \langle \hat{X} \hat{P} \rangle \) for a state that is \( |\psi(t = 0)\rangle = \frac{i}{\sqrt{3}}|0\rangle + \sqrt{\frac{2}{3}}|1\rangle \). This is a function of time.

(3) Coherent State. \( |\psi(0)\rangle = N(|z_0 > -i | - z_0 \rangle) \), where \( z_0 \) is a complex number and:

\[
|z\rangle = \exp(-|z|^2/2) \exp(z a^\dagger)|0\rangle.
\]

(a) Determine \( N \) by normalizing the initial state. (b) Compute \( |\psi(t)\rangle \) and use it to find \( \langle X \rangle \) and \( \langle P \rangle \) as functions of time. Show the time-dependence explicitly and make sure that both results are real. Verify Ehrenfest's Principle, \( \langle P \rangle = m \frac{d}{dt} \langle X \rangle \). (c) Compute \( \langle X^2 \rangle \) and \( \langle P^2 \rangle \) as functions of time. (d) Use these results to find \( \Delta X \) and \( \Delta P \) and verify Heisenberg's Uncertainty Principle.