1) \( V(r) = V_0 \theta(a-r) \), a repulsive spherical potential. Assume that the energy, \( E = k^2/(2m) \), is greater than the barrier, \( E > V_0 \). Compute the \( l = 0 \) phase shift, \( \delta_0 \), and the cross section from \( l = 0 \) scattering. Try to find the limit as \( E \to V_0 \). Compare your result to what you get using the Born approximation and comment.

2) Use the Born approximation to estimate the total cross-section, \( \sigma \), for \( V(r) = \alpha \delta(a-r) \).

3) Use the \( l = 0 \) partial wave only, computing the phase shift for a repulsive potential \( V(r) = V_0 \theta(a-r) \). Assume the energy is less than \( V_0 \), so that the potential is classically “forbidden”. Given \( \delta_0 \), what is the cross section for \( E = k^2/(2m) \) when \( E < V_0 \), and what is the limit as \( k \to 0 \)?

4) Three identical spin-1/2 fermions are confined to a two-dimensional box, \( 0 < x < a \) and \( 0 < y < a \). What are the three lowest energies, their degeneracies and their eigenstates?

5) For the system in problem 4, a repulsive potential \( V = \alpha [\delta(x_1 - x_2) + \delta(x_1 - x_3) + \delta(x_2 - x_3)] \) is added to the original hamiltonian, \( H_0 \). What are the first order energy shifts for the ground state and first excited state?